Paper accepted for oral presentation at: International Conference on High Performance Structures and Composites. March 2002, Seville, Spain. Wessex Institute of Technology, UK.

An energy formulation for the passive control of the seismic response of buildings

Julio C. Miranda
Structural Engineering Department
IDC, United States of America

Abstract

This paper presents the mathematical model of an energy-formulated approach for the passive control of buildings isolated at several levels. Viscous energy dissipating devices are also inserted in parallel with the isolators. These levels may occur naturally, such as at the base of setbacks, the base proper, or may be selected so as to obtain a desired seismic performance.

With that aim, using modal analysis the present paper formulates the response of buildings idealized as two degree of freedom systems, in terms of the kinetic energy imparted upon each mass. The resulting equations provide the corresponding modal properties such as mode shapes, frequencies, damping, and participation factors, in a manner easy to understand and rationalize. These results are applicable to the passive control of the seismic response, in the measure that the isolation and damping devices are manufactured to correspond to the selected values.

It is hoped that this paper presents an innovative way of thinking in the manner in which engineers can control the behavior of their designs. It extends the range of applications thus far envisaged for state-of-the-art protecting devices such as isolators and dampers.

1 Introduction

Research on the use of laminated steel-rubber isolators for the seismic protection of buildings began in the early 1970s at the Laboratoire de Mécanique et d'Acoustique of the Centre National de la Recherche Scientifique (CNRS) in Marseille, France [7]. A number of structures in France and California , including nuclear facilities, have to date been provided with the isolation system developed therein. Since that time, researchers in England, Italy, Japan, New Zealand, and the United States have developed other types of base isolation systems which are being implemented in a variety of projects. Regulations for the implementation of base isolation have been included in important codes such as the International Building Code [1].

The ending of the Cold War at the end of last century, gave impetus to the research and application to civilian life of energy dissipating techniques that had previously been applied mainly within a military context. In the United States the bulk of this work has been carried out at the University of Buffalo, New York, and the University of California at Berkeley. A growing number of buildings in Japan, Italy, and the United States have been provided with such devices. Although not codified yet, this work has lead to the publication by NHERP [2], of suggested design guidelines.

Inspired by this state of progress, the present paper intends to explore the passive control of the structural seismic response of buildings based on its energy aspects. Linearly elastic isolators plus linear viscous dampers are placed at different elevations of buildings. As shown on figure 1 some of these levels are obvious, such as the base-ground interface for traditionally base isolated buildings, or at the setback-building interface of building with setbacks, figures 1.a and 1.c respectively. More generally, these levels may be pre-selected so as to achieve a contemplated level of control, figure 1.d. The buildings are idealized as two degree of freedom systems within a modal analysis context. A number of expressions are provided which furnish the mode shapes, frequencies, damping, and participation factors of the structures to be controlled. It is expected that the study of these expressions, coupled with numerical analysis, will lead to a better understanding of the passive control capabilities of these devices, and later to some real life applications.

2 Theoretical development general case

Each of the buildings shown on figure 1 can be assimilated to the conceptual model shown on figure 2, wherein the upper portion of the structure is characterized by its generalized mass M_U, its generalized damping C_U, and its generalized horizontal stiffness K_U. The lower portion of the structure is similarly characterized by its generalized mass M_L, its generalized damping C_L, and its generalized horizontal stiffness K_L. It can be shown that the equation of motion for the model depicted in figure 2, when excited by a horizontal earthquake with acceleration *a(t)*, is:

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = -Mra(t)$$
(1)

where *r* is a vector of ones, and *y(t)* is the vector of horizontal displacements relative to the ground. The displacement components for the upper and lower masses are y_u and y_l respectively. *M*, *C*, and *K* are the mass, damping, and lateral stiffness matrices of the system respectively, and are given by:

$$M = \begin{bmatrix} M_U & 0 \\ 0 & M_L \end{bmatrix}$$ (1.a)

$$K = \begin{bmatrix} K_U & -K_U \\ -K_U & K_U + K_L \end{bmatrix}$$
(1.b)

$$C = \begin{bmatrix} C_U & -C_U \\ -C_U & C_U + C_L \end{bmatrix}$$
(1.c)

If we assume that the system under consideration is classically damped, the frequencies and mode shapes may be obtained by solving the eigenproblem:

$$\lfloor K - w_i^2 \; M \rfloor X_i = 0$$
(2)

where w_i is the circular frequency of the system for the corresponding mode shape X_i. Prior to solving this equation, let us define the following variables:

$$w_L^2 = \frac{K_L}{M_L}$$ (3.a)

$$w_U^2 = \frac{K_U}{M_U}$$
(3.b)

$$\Omega = \frac{w_U}{w_L}$$ (3.c)

$$m = \frac{M_U}{M_L}$$
(3.d)

$$a_i = \frac{M_U X_{Ui}^2}{M_L X_{Li}^2}$$
(3.e)

Equation (3.a) provides the circular frequency for the lower portion of the structure when considered independently, and as having a single horizontal degree of freedom. Similarly, Equation (3.b) provides the circular frequency for the upper portion considered independently, and with a single horizontal degree of freedom. Equation (3.c) represents a frequency ratio between the independently considered upper and lower portions. Equation (3.d) is the ratio of the upper mass to the lower mass. Equation (3.e) represents the ratio of the kinetic energy assumed in a given mode X_i by the upper and lower masses respectively.

If arbitrarily the modal displacement components at the lower mass are made equal to one, from equation (3.e) we have:

$$X_{Ui} = \pm\sqrt{\frac{a_i}{m}}$$
(4.a)

Hence, the mode shapes can be written as:

$$X_1 = \left[\sqrt{a_1 / m} \qquad 1 \right]^T \qquad (4.b) \qquad X_2 = \left[-\sqrt{a_2 / m} \qquad 1 \right]^T$$

(4.c)

Replacing the mode shapes in equations (4.b) and (4.c) into equation (2) results in the circular frequencies:

$$w_1^2 = \frac{w_U^2 (\sqrt{a_1} - \sqrt{m})^2 + w_L^2}{1 + a_1} \quad (5.a) \quad w_2^2 = \frac{w_U^2 (\sqrt{a_2} + \sqrt{m})^2 + w_L^2}{1 + a_2} \quad (5.b)$$

In view of the orthogonality of the mode shapes with respect to the mass and stiffness matrices, the following expressions are to be complied with :

$$a_1 a_2 = 1 \qquad (6.a) \qquad \Omega^2 = \frac{1}{1 + \sqrt{m a_1} - \sqrt{m a_2} - m}$$

(6.b)

Let us recall that the modal participation factors are written:

$$g_i = \frac{X_i^T M r}{X_i^T M X_i}$$

(7) where the subscripts denote the order of the mode under consideration. After replacement of the appropriate values we obtain:

$$g_1 = \frac{1 + \sqrt{m a_1}}{1 + a_1} \qquad (8.a) \qquad g_2 = \frac{1 - \sqrt{m a_2}}{1 + a_2}$$

(8.b)

To calculate the modal damping we need to recall that the generalized damping for each mode is written, [8]:

$$X_1^T C X_1 = 2 x_1 w_1 X_1^T M X_1 \quad \text{and} \quad X_2^T C X_2 = 2 x_2 w_2 X_2^T M X_2$$

Similarly for the upper and lower portions considered independently:

$$C_U = 2 x_U w_U M_U \qquad \text{and} \qquad C_L = 2 x_L w_L M_L$$

After the appropriate substitutions in the above equations we find the following expressions for the overall modal damping coefficients:

$$x_1 = A x_U + B x_L$$

(9.a)

$$x_2 = C x_U + D x_L$$

(9.b)

where: $$A = \frac{\Omega(\sqrt{a_1} - \sqrt{m})^2}{(\sqrt{1+a_1})(\Omega^2(\sqrt{a_1} - \sqrt{m})^2 + 1)^{1/2}}$$

(9.c)

$$B = \frac{1}{(\sqrt{1+a_1})(\Omega^2(\sqrt{a_1} - \sqrt{m})^2 + 1)^{1/2}}$$

(9.d)

$$C = \frac{\Omega(\sqrt{a_2} + \sqrt{m})^2}{(\sqrt{1+a_2})(\Omega^2(\sqrt{a_2} + \sqrt{m})^2 + 1)^{1/2}}$$

(9.e)

$$D = \frac{1}{(\sqrt{1+a_2})(\Omega^2(\sqrt{a_2} + \sqrt{m})^2 + 1)^{1/2}} \qquad (9.f)$$

3 Particular case of perfectly base isolated buildings

Let us assume that the kinetic energy imparted upon each mass is directly proportional to itself. Under these conditions:

$a_1 = m$ (10.a) $a_{12} = 1/m$

(10.b)

and the mode shapes are:

$X_1 = [1 \quad 1]^T$ (11.a) $X_2 = [-1/m \quad 1]^T$

(11.b)

Equation (11.a) indicates that this behavior corresponds to a perfectly based isolated building, such as the one shown on figure 1.a. The idealization of the structure as a two degree of freedom system has been successfully used before, [3]. In this instance M_U represents the upper portion of the building, whereas M_L represents its base or first level. The frequencies are:

$w_1^2 = K_L /(M_U + M_L)$ (12.a) $w_2^2 = w_U^2 (1+m)$

(12.b)

The modal damping coefficients are:

$$x_1 = \frac{x_L}{\sqrt{1+m}} \quad (13.a) \qquad x_2 = (\sqrt{1+m})x_U$$

(13.b)

and finally the participation factors are:

$$g_1 = 1 \quad (14.a) \qquad g_2 = 0$$

(14.b)

These expressions match those derived through other procedures, see [9] for example.

4 Particular case of highly damped tuned masses

Observation of the mode shapes in equations (4.b) and (4.c), indicates that if perfect energy transfer is achieved between the masses, and the upper mass is small enough, the displacement of this mass may be quite important. This provides us with a mechanism to impose high over all damping through a highly damped tuned upper mass. The schematic representation of the building would correspond to figure 1.b. For perfect energy transfer we have:

$$a_1 = 1 \quad (15.a) \qquad a_{12} = 1$$

(15.b)

and the mode shapes are:

$$X_1 = \begin{bmatrix} 1/\sqrt{m} & 1 \end{bmatrix}^T \quad (16.a) \qquad X_2 = \begin{bmatrix} -1/\sqrt{m} & 1 \end{bmatrix}^T$$

(16.b)

Equation (6.b) is now written:

$$\Omega^2 = \frac{1}{1-m}$$

(17)

Resonance can only be satisfied if m is zero in the above equation. In what follows we will assume that the upper mass is very small in relation to the lower mass, such that in the limit $m \rightarrow 0$. The frequencies can now be written:

$$w_1 = w_L(1 - m^{1/2}/2) \quad (18.a) \qquad w_2 = w_L(1 + m^{1/2}/2)$$

(18.b)

The modal damping coefficients are:

$$x_1 = Ax_U + Bx_L \tag{19.a}$$

$$x_2 = Cx_U + Dx_L \tag{19.b}$$

where:

$$A = \frac{(1 - m^{1/2})^2}{2 - m^{1/2}} \tag{19.c}$$

$$B = \frac{1}{2 - m^{1/2}} \tag{19.d}$$

$$C = \frac{(1 - m^{1/2})^2}{2 + m^{1/2}} \tag{19.e}$$

$$D = \frac{1}{2 + m^{1/2}} \tag{19.f}$$

Finally the participation factors are:

$$g_1 = \frac{1 + m^{1/2}}{2} \tag{20.a}$$

$$g_2 = \frac{1 - m^{1/2}}{2} \tag{20.b}$$

Since the values of m are very small, equations (19) indicate that modal damping is approximately equal to the average of the addition of the upper portion damping and the lower portion damping. This means of passive control of the seismic response has been proposed before, [4],[5],and [6]. Some numerical studies, [5], for buildings fixed to the base and with a heavily damped tuned upper mass indicate a base shear reduction of about 30%, and lower mass displacement reduction of about 40%. A schematic representation of the tuned mass is shown on figure 2, where Gapec, and Domange Jarret, are the commercial names of the isolators and dampers considered in the numeral studies referred to above.

5 Multilevel isolated buildings

We consider here buildings that present setbacks, figure 1.c, or building that can purposely be provided with isolation at two levels, figure 1.d. Given the number of different parameters involved, the selection of the must suitable combination
for passive control is not unique, it is earthquake dependant, and a subject which undoubtedly deserves much additional research. The approach which we are to follow, consists in inducing resonance between the upper and lower masses, while having finite values of m . In this way, the transference of energy between the masses is not perfect, and while one of the masses may have a "surplus" of kinetic energy, the other may have a "deficit". However, resonance will maximize energy transfer as much as possible. We regard the lower mass as the mass to be controlled, and for illustration, let us specialize the equations for a value of m equal to 0.25. Equation (6.a) and (6.b) yield: $a_1 = 1.640$ and $a_2 = 0.610$.
The mode shapes, equation (4.b) and (4.c) are written:

$$X_1 = [2.561 \quad 1]^T \qquad X_2 = [-1.562 \quad 1]^T$$

with the corresponding frequencies, equation (5.a) and (5.b):

$$w_1 = 0.781 w_L \qquad w_2 = 1.281 w_L$$

and participation factors, equation (8.a) and (8.b):

$$g_1 = 0.621 \qquad g_2 = 0.379$$

The damping coefficients are, equation (9.a) and (9.b):

$$x_1 = 0.296 x_U + 0.485 x_L \qquad x_2 = 0.796 x_U + 0.485 x_L$$

We notice the first apparent advantage of multilevel isolation; the frequency of the lower mass is lowered due to the isolation systems in series. The second

advantage is the accommodation of the displacement demand at several isolation
interfaces. This is particularly attractive for systems sensitive to P-Δ effects, such as for rubber-steel bearings, since the capacity is not provided at a single level. However, the second frequency is increased, and if increased enough it could fall at the peak of the response spectrum. This fact is countered by the reduced second mode participation factor, and by the enhanced second mode damping. Also, it is to be noted that the importance of the first mode damping is somewhat reduced. To illustrate this effect, if both isolation levels are capable of damping coefficients of 0.20, the overall damping coefficients are:

$$x_1 = 0.16 \qquad x_2 = 0.26$$

It might be desirable to provide a better control of the second mode. For example, the upper portion may be provided with a damping coefficient of 0.4, while still maintaining the lower portion damping coefficient at 0.2 . In this instance the overall damping coefficients are:

$$x_1 = 0.22 \qquad\qquad x_2 = 0.42$$

To conclude, it would seem clear that numerical studies, which would include the appropriate variety of spectra, are required in order to converge to the suitable combination of parameters, in a way that passive control of the seismic response of the building is satisfactorily achieved. It would also seem clear however, that such passive control is feasible, and achievable.

6 Conclusions

A simple mathematical model has been developed to calculate the passive control of the seismic response of buildings. The procedure is capable of predicting limit case behavior, previously found through alternate means, of base isolation, and tuned mass concept. Although more research in undoubtedly required, it is apparent that the kinetic energy formulated model provides a solid platform for the study of the passive control using multilevel isolation of buildings.

Disclaimer

The ideas and opinions expressed in this paper are solely the responsibility of the writer, and do not necessarily represent the policies and procedures of Industrial Design & Construction.

References

[1] International Code Council, *International Building Code*, 2000 Edition.

[2] Building Seismic Safety Council, *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures, 2000 Edition*, Part 1: Provisions, and Part 2: Commentary.

[3] Miranda J.C., *A Simplified Model for the Elastic Dynamic Behavior of Base Isolated Buildings*. Fourth World Congress on Joint Sealants and Bearing Systems for Concrete Structures, ACI SP-164, Sacramento, California, 1996.

[4] Villaverde R., Mosqueda G., *Architectural Detail to Increase the Seismic*

Resistance of Buildings. Structural Engineers World Congress, San Francisco, California, 1998.

[5] Miranda J.C., *Seismic Protection of Buildings with Passive Tuned Mass Dampers.* Structural Engineers World Congress, San Francisco, California, 1998.

[6] Shao D., Miyamoto K., *Seismic Retrofit of a Non-ductile Concrete Structure Using Heavily Damped Tuned Mass Devices.* Proceedings 69th Annual Convention, Structural Engineers Association of California, 2000.

[7] Delfosse G.C., Miranda J.C., *Buildings on Isolators for Earthquake Protection*. Proceedings of the Second International Conference on Microzonation for Safer Construction- Research and Application. San Francisco, California, 1978.

[8] Clough R.W., Penzien J., *Dynamics of Structures.* McGraw-Hill, Inc., 1975

[9] Miranda J.C., *Structural Dynamics of Base Isolated Buildings.* Experimental Center of Engineering, Faculty of Civil Engineering, Technological University of Panama, August 2000.

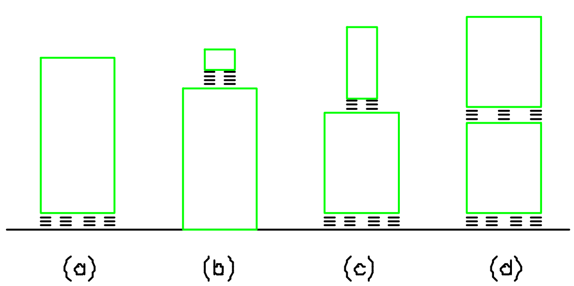
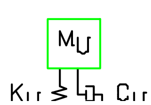


FIGURE 1



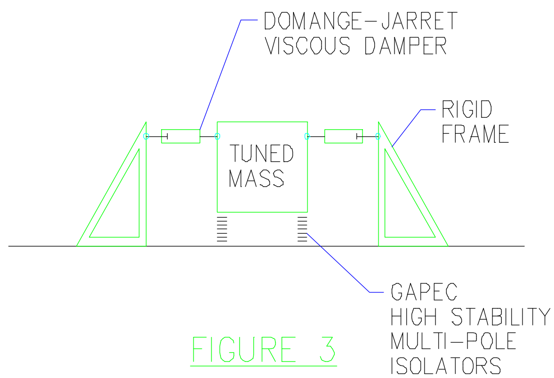

DOMANGE-JARRET
VISCOUS DAMPER
RIGID
FRAME
TUNED
MASS
GAPEC
HIGH STABILITY
MULTI-POLE
ISOLATORS

FIGURE 3